1904MNRAS..64..458C

of stars per square degree is 0·632 or (the difference of magnitude being 1·5) at the rate of 0·42 per magnitude.

Between 20^s and 6^m the log increase of number of stars per square degree is 0·757, or (the difference of magnitude being 2·4) at the rate of 0·32 per magnitude.

Taking the mean of these results we find that between the magnitudes 10^m·0 and 12^m·0 the logarithmic increase per magnitude in the total number of stars is 0·37, and therefore the ratio of increase per magnitude is 2·34.

Between 3^m and 40^m there is a logarithmic increase of 0·839, corresponding to a difference of magnitude of 3^m·0 or 0·28 per magnitude, and between 6^m and 40^m of ·713, corresponding to a difference of magnitude of 2^m·1 or of 0·34 per magnitude.

Taking the mean of these results we have between 12^m·0 and 14^m·4 a logarithmic increase of number of stars per magnitude =0·31, corresponding to a ratio of increase of 2·04 per magnitude.

It is satisfactory to find that these results are in good accordance with those recently given by Professor Pickering in his valuable memoir on the "Distribution of Stars" (*Harvard Annals*, vol. xlviii. No. 5, p. 178). Taking the values given in his Table XXI., the logarithmic increase per magnitude in the total number of stars would be 0·39 (as compared with 0·37 found above) between 10 and 12 mag., and 0·31 (as compared with 0·31) between 12^m·0 and 14^m·4, the corresponding ratios of increase per magnitude in the total number of stars being 2·45 and 2·04 found by Professor Pickering, as compared with 2·34 and 2·04 found above.

Royal Observatory, Greenwich:
1904 *March* 8.

On the Chromatic Correction of Object-glasses. Second Paper.

By A. E. Conrady.

In my first paper [*] I limited myself to the consideration of a luminous point in the optical axis. Precisely the same reasoning may, however, be applied to any point in the field of an object glass; but as in the case of oblique pencils there is no such complete symmetry as in axial pencils, a greater number of rays have to be followed through the system of lenses and their paths in the successive media determined in order to compare the value of $\Sigma dn\mathrm{D}$ for the several rays, and thus to determine the state of the chromatic correction.

It will generally be sufficient to compute four rays through the system: three of these lie in the plane containing the optical axis and the luminous point; these should be computed

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[*] *Monthly Notices*, January 1904.

1904MNRAS..64..458C

by the trigonometrical formulæ given in my previous paper. They are those rays in that plane which pass through the centre and margins of the limiting aperture of the system. The fourth ray passes through the margin of the same aperture vertically above or below the central ray; it does not cut the optical axis and must be computed by formulæ adapted to this case; probably those given by von Seidel * are the most convenient. The paths of each of these four rays in the several constituent lenses of the system can then be computed by the formula given in my previous paper (*l.c.* page 187). In the case of the fourth ray, and adopting the nomenclature used in Steinheil and Voit's reprint of Seidel's classical paper, the angle at the centre of curvature, which I call a, is there given as η, and the angle between the ray and a parallel to the optical axis, which takes the place of my angle β', is also given as τ', so that my formula for D may be applied with the same facility to this ray as to those proceeding in planes containing the optical axis.

Then, if $\Sigma dn \, . \, \mathrm{D}$ has exactly the same value for all four rays, the system will be perfectly achromatic for the luminous point chosen and for the selected wave-length. But the values of the sum will generally, owing to the extreme difficulty of perfect correction for oblique pencils, differ more or less from each other. Let $\Sigma dn\mathrm{D}_0$ be the value of the sum for the central ray, $\Sigma dn\mathrm{D}_1$ and $\Sigma dn\mathrm{D}_2$ the respective values for the marginal rays in the plane containing the optical axis, and $\Sigma dn\mathrm{D}_3$ the value for the marginal rays farthest from that plane; then it is not difficult to see that:

1. If $\Sigma dn\mathrm{D}_1 \gtrless \Sigma dn\mathrm{D}_2$ the refracted coloured waves will form an angle with, and will therefore proceed in a direction differing from, that of the refracted principal waves; in other words, the object-glass will produce different-sized images for different colours, or the image of the luminous point will be drawn out into a spectrum.

2. If $\Sigma dn\mathrm{D}_1 = \Sigma dn\mathrm{D}_2$ but $\gtrless \Sigma dn\mathrm{D}_0$, the coloured and principal waves will have the same direction, but different curvature and therefore different foci, and the image of the luminous point will be affected with symmetrically arranged coloured fringes.

3. If $\Sigma dn\mathrm{D}_3$ differs from $\frac{1}{2}(\Sigma dn\mathrm{D}_1 + \Sigma dn\mathrm{D}_2)$ there will be an astigmatic difference for different colours, and similar considerations will lead to a correct interpretation of any other case that may be met with.

The reasoning employed in my first paper may also be applied to prisms, for there is nothing in the train of thoughts which leads to my expression for the chromatic aberration of a system of lenses which cannot be equally well applied to plane or curved waves proceeding through a train of prisms; hence if d

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* "Sitzungsberichte der math.-phys. Classe der kgl. bayr. Akademie der Wissenschaften vom 10. Nov. 1866," reprinted in Steinheil and Voit, *Handbuch der angewandten Optik*, vol. i., Leipzig, 1891.

1904MNRAS..64..458C

and D are taken as the respective paths of the two marginal rays transmitted, $\Sigma dn(d-D)$ expresses the differential dispersion of the train of prisms, measured as the distance between the refracted principal and coloured waves on one ray when they intersect on the other; and accepting the postulate that this distance must be equal to the wave-length in order to give sufficiently separated foci of the two waves, we obtain at once Lord Rayleigh's famous formula for the resolving power of spectroscopes,* at which the great physicist arrived by a totally different path.

Conversely, my formula as applied to lenses may be interpreted in accordance with the familiar treatment of lenses as prisms of variable angle as demanding that the total resolving power of the system shall be zero for the selected wave-length, and so stated it has an almost axiomatic appearance.

With regard to suitable formulæ for determining the refractive and dispersive powers of optical glass for any wave-length, I have repeated the necessary calculations, with the result that the statements in my first paper are amply confirmed; it would indeed seem that the formula there given as the best:

$$n_\lambda = n_0 + \nu_1'\lambda^{-\frac{9}{7}} + \nu_2\lambda^{-4}$$

is even more accurate than I stated, and will generally represent the indices of any ordinary silicate glass within the visible spectrum to less than one unit of the fifth decimal if well selected lines are chosen for determining the constants.

These calculations led to the discovery of yet another empirical formula which is slightly more convenient and at least equally accurate, and should therefore be preferable, viz.

$$n_\lambda = n_0 + \nu_1\lambda^{-1} + \nu_2\lambda^{-3\frac{1}{2}}$$

Finally, an application of the various formulæ to Langley's determination of the refractive indices for a light flint prism from λ = 20,300 to λ = 3440 (O line) had the interesting result of showing that the empirical formulæ retain their superiority throughout that range, for when the constants were determined from the indices for the A, F, and O lines, Cauchy's formula (with three terms) erred by twice as much for the longest wave-length as any of the others, and for other wave-lengths the superiority of the empirical formulæ proved even greater.

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* *Encyc. Brit.*, 9th edition, vol. xvii. page 807. Also "Investigations in Optics," *Phil. Mag.* 1879, 1880.

Bedford Park, W.: 1904 *March* 10.

1904MNRAS..64..458C

On the Determination of the Division Errors of a Graduated Circle. By S. S. Hough, M.A., F.R.S.

§ 1. *Introductory.*

The accurate determination of the division errors of a graduated circle can only be made at the expense of a considerable amount of labour, and it becomes a matter of the highest importance at the outset of such an undertaking to investigate how the available appliances may be utilised to the highest advantage with the least possible work. The considerations dealt with in the following paper have arisen in the course of the preparation of a programme for the determination of the errors of divisions of the new transit circle of the Cape Observatory now in progress, and it will serve to coordinate ideas if we deal with the problem from the outset in the form in which it is presented by that instrument. The subsidiary appliances will, of course, vary with different instruments, but those attached to the Cape instrument may at least be taken as sufficiently typical, while the principles involved may be readily adapted to other instruments and appliances.

The circles of the Cape instrument are ordinarily read by means of six microscopes symmetrically situated round the centres. For the determination of errors of graduation the instrument is also provided with two pairs of subsidiary microscopes, situated each at extremities of a diameter at angular distances of 20° and 25° respectively from one pair of the primary six.

The six primary microscopes enable us to determine the relative errors of six equi-spaced divisions with a high degree of accuracy, and the subsidiary microscopes then furnish the means of determining the errors of division—first, of division marks separated from the former by an interval of 20°; and, secondly, of those separated by an interval of 25°—*i.e.* we can ascertain the relative spacing of division marks separated by intervals of 5° throughout the circle. A further appliance, consisting of a microscope provided with a divided object glass similar to that used in the determination of the division errors of the old Cape transit circle,* enables us still further to subdivide the spaces of 5° into spaces of 1° interval.

We proceed first to consider the geometrical significance of a single pointing made by means of one of the micrometers on a division mark.

§ 2. *Derivation of the Fundamental Equation.*

Assume, in the first instance, that the circles are perfectly plane and perfectly rigid, and that the division marks consist of approximately radial straight lines engraved on them.

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* Introduction to *Cape Observations*, 1879–81.